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BRIEF COMMUNICATION

THE THEORY OF THE GAS-LIFT PUMP

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1. INTRODUCTION

A theory has been developed recently for one area of gas–liquid two-phase flow, namely that of the gas-lift pump (Husain 1975). In this model the gas-lift pump is viewed as a closed thermodynamic system composed of inviscid gas- and liquid-fluids with insufficient internal energy, that is potential energy, in the downcomer to raise the liquid up the riser without the injection of some external energy. The required energy is assumed to be supplied by the isothermal expansion of the gas in the riser. The riser acts as a self-contained zone of gas–liquid mixture whose mean density is governed by the superficial gas velocity at the orifice plane, but within which there exists also a basic liquid circulatory motion characterised by a constant effective bubble rise velocity. Thus, the pump acts as an inverted syphon operating under conditions of mean physical equilibrium at all times while the orifice is equivalent to an emitter of constant frequency determined by the gas flow rate but emitting over a range of power levels obeying the statistical distribution law for quantisized systems. The system is analogous to that involved in the theory of black body radiation. The average power output of the orifice emitter so derived is combined with relationships derived for the growth of bubbles in the inviscid liquid to give an output equation.

2. THEORY

Husain (1975) has shown that the average energy relationship of an orifice plane emitting at a given frequency is

$$\overline{\epsilon_b} = E/N = e_0/[\exp(e_0/\epsilon_m) - 1]. \qquad [1]$$

where $\overline{\epsilon_b}$ is the average energy rate per emission of the orifice, ϵ_m the mean emission power, e_0 the quantum energy rate, E the total energy rate and N the total number of emissions or bubbles from the orifice plane.

Equation [1] gives the internal energy of the bubbles emitted by the orifice of the gas-lift pump. To obtain the energy input, it is necessary to know the bubble frequency for the system. Observations of a gas-lift pump in operation shows that two distinct bubble regimes exist in the riser. In the region of the orifice plane spherical bubbles exist. For this region the spherical bubble frequency relative to the orifice is

$$N_1 = \left[\frac{4\pi(U_L')^3}{3}\right]^{0.5} \frac{1}{V_G^{0.5}}, \qquad [2]$$

and the spherical bubble volume is given by

$$V_{b_1} = V_g' = \left[\frac{3}{4\pi(U_L')^3}\right]^{0.5} V_G^{0.5}, \qquad [3]$$

where U_L' is the recirculation velocity, V_G the gas volumetric flow rate and t the time. The remainder of the riser involves cylindrical bubbles of the slug flow type produced in the moving stream by coalescence of the spherical bubbles from the orifice region.

The transformation from spherical to slug type bubbles takes place in the moving stream. Thus it is necessary to postulate a notional slug flow emitter, moving with the liquid whose frequency is related to the orifice by an equation of continuity, in order to obtain the true energy density relative to the riser. This leads to the cylindrical bubble frequency being

$$N_2 = \frac{gR_a}{4\beta u_G}, \tag{4}$$

and the cylindrical bubble volume

$$V_{b_2} = V_G t = \frac{4\beta}{\pi r_b^2 g} V_G^2, \tag{5}$$

where g is gravitational acceleration, R_a the ratio of the bubble cross-section to the channel cross-section to the channel cross-section, β the ratio of virtual mass to displaced mass, u_G the superficial gas velocity and r_b the bubble radius.

At constant operation the mean static pressure must be equal to ϵ_m the mean power output of the orifice emitter:

$$\epsilon_m = \rho_m hg V_G = \rho_L R_s hg V_G, \tag{6}$$

where ρ_m is the mean density, ρ_L the liquid density, R_s the submergence ratio which is the ratio of the submergence length hg to the riser length.

Defining the energy rate for an emission (i.e. a bubble)

$$e_0 = HhN_2 V_G, \tag{7}$$

where H is some constant per unit riser length, and substituting from [4] and since $V_G = Au_G$,

$$e_0 = K_1 hAg, \tag{8}$$

where K_1 is a constant and A is the cross-section of the conduit. Placing [6] and [8] into [1] gives

$$\bar{\epsilon}_b = K_1 hAg \Big/ \left[\exp\left[\frac{K_1 V}{\rho_L R_s V_G}\right] - 1\right]. \tag{9}$$

Equation [9] defines the average rate of energy transport per bubble of volume V_{b_2} and frequency N_2, relative to the liquid. However, the time frequency relative to the riser is the orifice frequency N_1 and its corresponding bubble volume V_{b_1}.

Thus the number of bubbles in the riser is

$$\frac{\alpha_2 hA}{V_{b_1}}, \tag{10}$$

where α_2 is the riser gas volume fraction, and the average gas energy input rate per bubble becomes

$$E_G = \frac{\alpha_2 hA}{V_{b_1}} \bar{\epsilon}_b. \tag{11}$$

Before [11] can be used it is necessary to be able to evaluate the bubble volume V_{b_1} or to define the unknown velocity U'_L. After considerable work it was assumed that U'_L was a nett recirculation velocity, characteristic of the statistical system set up within the riser of given dimensions. This inevitably led to the conclusion that the riser apparently acts as a self-contained zone having an effective overall pressure gradient, determined by the gas flow rate or voidage, through which the output liquid flows at a rate determined by the balance of riser and downcomer forces at the orifice plane. Thus U'_L is defined by the equation

$$U'_L = \left(\frac{2gA}{h}\right)^{0.5}, \tag{12}$$

which suggests the interesting proposition that a two-phase statistical system enclosed in a channel can be characterised by a surface tension.

Substitution of [12] in [13] gives

$$V_{b_1} = \left(\frac{3}{4\pi}\right)^{0.5}\left(\frac{h}{2gA}\right)^{0.75}\left(\frac{W_G}{\rho_{G0}}\right)^{1.5}, \tag{13}$$

where W_G is the mass flow rate of the gas and ρ_{G0} is the density of the gas at orifice plane. The energy rate required to raise the liquid output is

$$E_L = W_L gH, \tag{14}$$

$$= \alpha_1 E_G, \tag{15}$$

where α_1 is the efficiency of gas usage and W_L is the mass of flow rate of liquid. Substituting for E_G from [11], E_L from [14], $\overline{\epsilon_b}$ from [9] and V_{b_1} from [13] into [15] gives

$$W_L = \frac{K_2 A^{2.75} h^{0.25} \rho_{G0}^{1.5}}{W_G^{1.5}(e^x - 1)}, \tag{16}$$

where

$$x = K_1 A \rho_{G0} / \rho_L R_s W_G, \tag{17}$$

and the constants K_1 and K_2 are

$$K_1 = HR_a / 4\beta, \tag{18}$$

$$K_2 = 603.35 \alpha_1 \alpha_2 K_1. \tag{19}$$

When applying [16] to [19] to real fluids, it is important to realise that the output characteristic will be sensitive to downcomer pressure drop and that an effective submergence ratio R_{seff} will occur which is different from the actual geometrical value. The equation applies

$$R_{seff} = R_s - \frac{2\Delta P_d}{h} \tag{20}$$

$$= (1 - \alpha_2) + \frac{\Delta P_{r_1}}{h}, \tag{21}$$

where ΔP_d is the potential head in the downcomer and ΔP_{r_1} is the head loss in the riser for zero frictional loss in the downcomer.

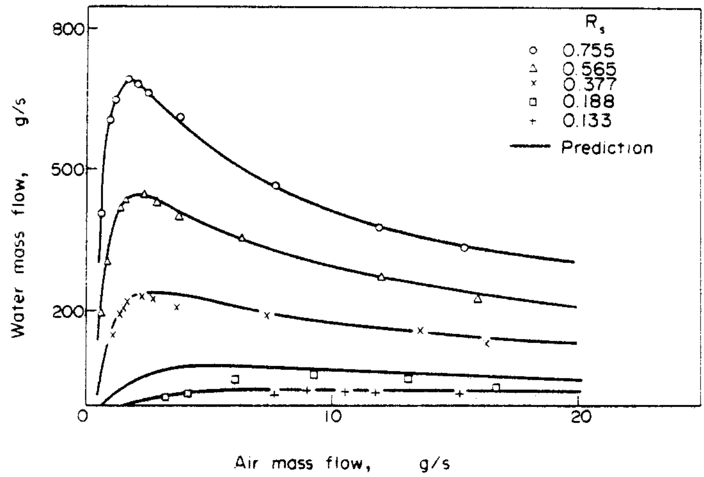

Figure 1. Air-lift pump experimental data of Gosline (1936) compared to the predictions of [16]. The system is air–water in a vertical riser 2.67 cm dis, 8.14 m long at 294°K and $1.013 \times 10^5 N/m^2$ absolute pressure.

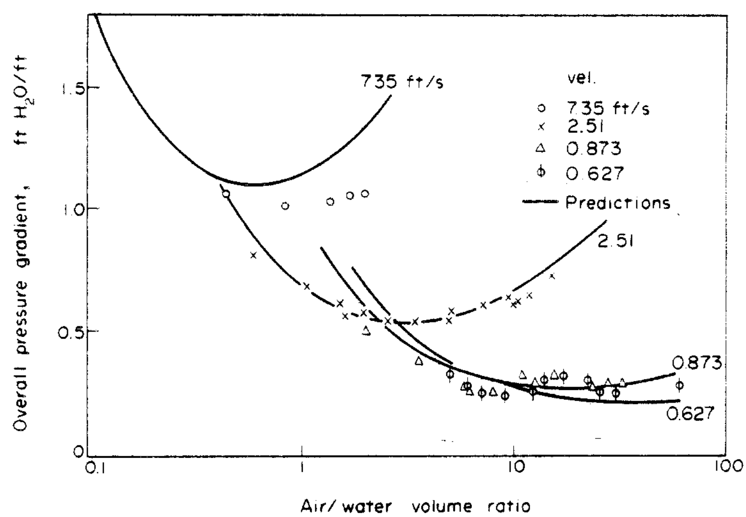

Figure 2. Comparison of the predictions of [16] for overall two-phase pressure drop with the data of Govier (1957). The system is air–water in a vertical riser 2.6 cm dia, 10.39 m long at 293°K and $2.48 \times 10^5 N/m^2$ absolute pressure.

3. EXPERIMENTAL

Data reported on the air lift pump by Gosline (1936), Govier *et al.* (1957) and Husain (1975) have been applied to [16] and have given the following values for the constants; $K_1 = 105$ g/s cm^2, $K_2 = 4.21 \times 10^4$(g/s cm^2)(cm/s^2)$^{0.75}$ and $\alpha_1\alpha_2 = 0.663$. Optimum values of various parameters were calculated for the condition of maximum isothermal efficiency. These optimum conditions were $x = 2.232$ and $R_s = 0.71$. The latter value agrees with the optimum submergence found experimentally from the data of Gosline (1936) and Nicklin (1963).

Figure 1 compares the output of an air-lift pump as reported by Gosline (1936) with the predictions of [16] indicating that agreement is reasonable between theory and experimental data. A more revealing comparison between [16] and the experimental data of Govier *et al.* (1957) is presented in figure 2 where the two-phase pressure drop is presented. Agreement between theory and experiment is reasonable except at the highest water velocity. Further experimental work is necessary in order to check out this discrepency. It is interesting to note how the predicted curves are tangent to an envelope given by the Lockhart–Martinelli (1949) type correlation.

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